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(DEPARTMENT OF PURE MATHEMATICS)

ZN 80/78 JANUARI

A.E. BROUWER

EMBEDDING THE AFFINE PLANE OF ORDER 4 IN A LINEAR SPACE WITH LINES OF SIZE 4 AND 85 POINTS

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Embedding the affine plane of order 4 in a linear space with lines of size 4 and 85 points

by

A.E. Brouwer

ABSTRACT

We answer a question of professor H. Lenz.

KEY WORDS & PHRASES: *embedding, linear space, block design.*

In [1] professor Lenz proves that for $v \equiv 1$ or $4 \pmod{12}$, $v \geq 49$ there exists a linear space on v points with lines of size 4 and a subspace of 16 points, with the possible exeptions of $v = 49$ and $v = 85$. Here we show that also in these cases such a space exists. (The notation the the usual Hanani-like one.)

<u>PROPOSITION 1</u>. *There exists for* $t \in \mathbb{N}$ *a* $B(\{4,(3t+1)^*\},1;9t+4)$ *design. In particular if* $t \equiv 0$ *or* $1 \pmod 4$ *then there exists a* $B(4,1;9t+4)$ *with a subspace of size* $3t + 1$. *In particular there exists a* $B(4,1;49)$ *with a subspace of size* 16.

<u>PROOF</u>. Ray Chaudhuri & Wilson proved the existence of a Kirkman triple system on $6t + 3$ points. Completing the $3t + 1$ parallel classes of this design with points at infinity yields the first statement; observing that Hanani proved that $u \in B(4,1)$ iff $u \equiv 1$ or $4 \pmod{12}$ yields the second one. Now take $t = 5$. □

<u>PROPOSITION 2</u>. *There exists a* $B(\{4,5\},1;28)$ *with a subspace of size* 5. *Consequently there exists a* $B(4,1;85)$ *with a subspace of size* 16.

<u>PROOF</u>. The implication is well known (and probably due to Hanani) so we only have to prove the first statement. Let $X = I_4 \times \mathbb{Z}_7$ and take the 21 quintuples

$$\{(0,0),(0,1),(1,3),(2,5),(3,4)\} \quad \text{mod } (-,7)$$
$$\{(0,0),(0,2),(1,6),(2,3),(3,1)\} \quad \text{mod } (-,7)$$
$$\{(0,0),(0,4),(1,5),(2,6),(3,2)\} \quad \text{mod } (-,7)$$

and the 28 quadruples

$$\{(0,0),(1,0),(2,0),(3,0)\} \quad \text{mod } (-,7)$$
$$\{(1,1),(1,2),(1,4),(2,0)\} \quad \text{mod } (-,7)$$
$$\{(2,3),(2,5),(2,6),(3,0)\} \quad \text{mod } (-,7)$$
$$\{(3,3),(3,5),(3,6),(1,0)\} \quad \text{mod } (-,7). \quad □$$

REFERENCE

[1] LENZ, H. *Embedding block designs into larger ones*, Preprint nr. 45, "Kombinatorische Mathematik", Freie Universität, Berlin, Aug. 1977.