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AFDELING ZUIVERE WISKUNDE (DEPARTMENT OF PURE MATHEMATICS)

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JANUAR1

A.E. BROUWER

EMBEDDING THE AFFINE PLANE OF ORDER 4 IN A LINEAR SPACE WITH LINES OF SIZE 4 AND 85 POINTS

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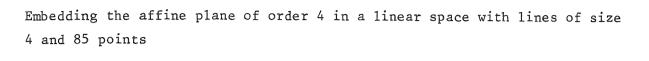
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EMBEDDING THE AFFINE PLANE OF ORDER 4 IN A LINEAR SPACE WITH LINES OF SIZE 4 AND 85 POINTS

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bу

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ABSTRACT

We answer a question of professor H. Lenz.

KEY WORDS & PHRASES: embedding, linear space, block design.

In [1] professor Lenz proves that for $v \equiv 1$ or 4 (mod 12), $v \ge 49$ there exists a linear space on v points with lines of size 4 and a subspace of 16 points, with the possible exeptions of v = 49 and v = 85. Here we show that also in these cases such a space exists. (The notation the the usual Hanani-like one.)

PROPOSITION 1. There exists for $t \in \mathbb{N}$ a $B(\{4,(3t+1)^*\},1;9t+4)$ design. In particular if $t \equiv 0$ or $1 \pmod 4$ then there exists a B(4,1;9t+4) with a subspace of size 3t + 1. In particular there exists a B(4,1;49) with a subspace of size 16.

<u>PROOF.</u> Ray Chaudhuri & Wilson proved the existence of a Kirkman triple system on 6t + 3 points. Completing the 3t + 1 parallel classes of this design with points at infinity yields the first statement; observing that Hanani proved that $u \in B(4,1)$ iff $u \equiv 1$ or $4 \pmod{12}$ yields the second one. Now take t = 5.

PROPOSITION 2. There exists a $B(\{4,5\},1;28)$ with a subspace of size 5. Consequently there exists a $B(\{4,5\},1;85)$ with a subspace of size 16.

<u>PROOF</u>. The implication is well known (and probably due to Hanani) so we only have to prove the first statement. Let $X = I_4 \times \mathbb{Z}_7$ and take the 21 quintuples

$$\{(0,0),(0,1),(1,3),(2,5),(3,4)\} \mod (-,7)$$

 $\{(0,0),(0,2),(1,6),(2,3),(3,1)\} \mod (-,7)$
 $\{(0,0),(0,4),(1,5),(2,6),(3,2)\} \mod (-,7)$

and the 28 quadruples

$$\{(0,0),(1,0),(2,0),(3,0)\}$$
 mod $(-,7)$
 $\{(1,1),(1,2),(1,4),(2,0)\}$ mod $(-,7)$
 $\{(2,3),(2,5),(2,6),(3,0)\}$ mod $(-,7)$
 $\{(3,3),(3,5),(3,6),(1,0)\}$ mod $(-,7)$.

REFERENCE

[1] LENZ, H. Embedding block designs into larger ones, Preprint nr. 45, "Kombinatorische Mathematik", Freie Universität, Berlin, Aug. 1977.